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THE EFFECTS OF LATERAL CONDUCTION ON HEAT FLUX ESTIMATION FROM SURFACE TEMPERATURE MEASUREMENTS

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ABSTRACT

Highly localized heating rates occur in extreme thermal environments, such as atmospheric reentry, and in experiments designed to model these environments, such as shock-shock interaction tests. Typically, temperature measurements are reduced to heat fluxes using one-dimensional conduction techniques. However, lateral conduction from localized high heat flux regions into low heat flux regions is significant and influences the one-dimensional solution. The solution also suffers mathematical instabilities. To evaluate the non-uniform, unsteady surface flux from measured temperatures, an inverse technique was devised that damps instabilities in the temporal direction and resolves large and sudden flux changes in the spatial direction. Based on previous work, a simple inverse method was used in time and a function specification method was used in space. Furthermore, a technique was devised to expedite the solution by "marching" in space as well as in time. The new multi-dimensional inverse method was found to more accurately resolve steep spatial gradients in flux than a one-dimensional method. Furthermore, the inverse procedure exhibits better stability than a multi-dimensional forward technique.

NOMENCLATURE

$\mathbf{H}$	order of regularization matrix (dimensionless)
q	heat flux (W/cm^2)
R	regularization term
S	objective function (K^2)
t	time (sec)
T	calculated temperature (K)
$\mathbf{X}$	sensitivity matrix
Y	measured temperature (K)
α	regularization parameter (K^2cm^4/W^2)
Ψ^{-1}	matrix of measurement variances ($1/K^2$)

Subscripts

i, j	temporal index
o	current iteration
s	space
t	time

INTRODUCTION

One aspect of space flight that must be considered during the design phase of aerospace vehicles is the propensity of a structure to destroy itself due to shock-shock interactions during high speed maneuvers. Damaging thermal environments can be created during high speed flight, and must be studied to prevent premature loss of flight. Examples of this baneful phenomenon have been noted on the X-15 (Burcham, Jr. and Nugent, 1970; Watts, 1968) and the space shuttle (Edney, 1970).

Shock-shock interaction studies have been performed for many years beginning with leading edge interference experiments (Hiers and Loubsky, 1967) and the characterization of shock interactions by Edney (1968). The significance of the potential damage and necessity for designs which account for these thermal phenomena has been the subject of numerous studies. For example, leading edge interference studies were of major interest because wings of high speed vehicles are prime candidates for shock interference heating. This work was continued by Wieting and Holden (1987), and later by Gladden and Melis (1994). The characterization of shock interaction heating was extended by Holden et al. (1988) who examined the different types of interactions relative to a cylindrical model (once again, to model a leading edge).

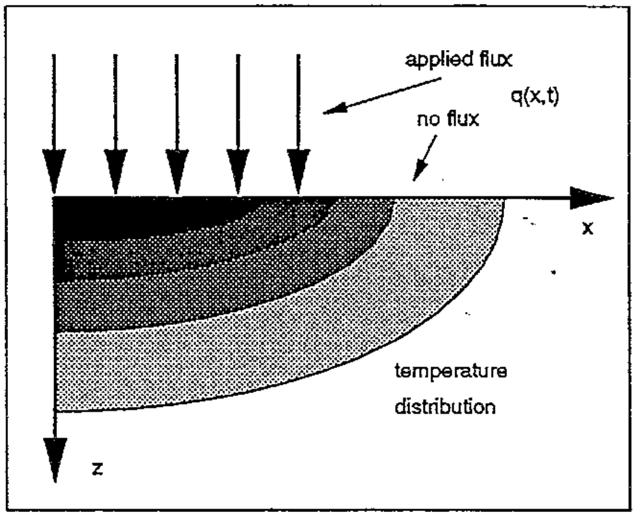

FIGURE 1: The temperature distribution due to a step flux (after some time) indicates that the temperature in the non-heated region will change.

Recently, more complex shock interactions have been studied such as those examined by Berry and Nowak (1996) and Neumann (1996).

One conclusion shared by all these experiments is that the heating generated by shock interaction is non-uniform in nature. In other words, the heating rates are extremely large and localized. Furthermore, oscillating levels of heat fluxes can be found at different positions on models subjected to shock interaction heating. The reader is referred to work by Edney (1968) and Glass et al. (1989) for an analysis of the flow patterns produced in shock interactions that are responsible for the anomalistic heating rates. Numerical work by Prabhu (1994) has since verified the findings of the nature of the heating rates found experimentally.

To evaluate the non-uniform heating rates, data reduction schemes have been used to convert measured temperatures from the surface of models to heat fluxes. This problem is unstable (Ehrich, 1954; Beck et al., 1985), and many methods have been proposed as solution methodologies. Walker and Scott (1997a) provides a summary of commonly used methods and their strengths and weaknesses. All currently used methods assume a one dimensional conduction model. However, a simple thought experiment can demonstrate the fallacy in this assumption.

Suppose a heat flux is being applied to a surface in a step fashion such that there is some area that sees a zero flux. We know that the heat being added will diffuse into the non-heated region causing the temperature to rise there as exemplified in Fig. 1. A one-dimensional data reduction routine, will result in a non-zero flux estimate for any sensor that sees a temperature change. Therefore, we must use a multi-dimensional conduction model to accurately describe a spatially dependent heating rate where the heat is diffusing laterally.

A direct solution to the data reduction would be to calculate the multi-dimensional conduction solution using the measured temperature as a boundary condition. The flux could then be found by differentiating the resulting temperature distribution (normal to the surface) at the surface and using Fourier's law. Direct methods, in general, have several inherent disadvantages as described by Walker and Scott (1995). For example, they tend to display unstable behavior resulting from differentiation of discrete data. Furthermore, we have to make some assumption about the temperature distribution between sensors and between time steps for these approaches. In effect, we are including high frequency information that can not possibly appear in discrete data which reduces our confidence in the solution methodology.

For an inverse approach we must first decide how the flux (not temperature) is to vary between sensors and between time steps. For most cases a linear change in flux between estimation points is assumed. The term "estimation points" is used here to mean the position and time where and/or when a flux is estimated. Normally this will coincide with the sensors and the measurements, however, we have suggested the potential to estimate fluxes with a higher spatial resolution than the actual measurements through an inverse technique. This approach was first examined by Walker and Scott (1997a).

The goal of this work is to examine the problem of lateral conduction effects in estimating heat fluxes from surface temperature measurements and to introduce a solution technique that can resolve fluxes that are spatially dependent. This work is a direct application of the methodologies discussed at a previous conference and described above (Walker and Scott, 1997b). Based on a previous study of the one-dimensional problem, we have chosen an inverse approach to solve problems where conduction effects are significant. A brief description of inverse solution techniques will be given as well as some results from an implementation of the inverse technology with experimental test data. These results will also be compared to a one-dimensional forward technique that is considered the state of the art (Hollis, 1995).

EXPERIMENT DESCRIPTION

The shock interaction temperature data used for this study was obtained from NASA Langley Research Center (LaRC); the test was documented by Berry and Nowak (1996). A three dimensional flow pattern was generated with a planar incident shock impinging a cylinder that was arranged perpendicular to the shock (see Fig. 2). The interaction between the planar shock and the bow shock produced hypersonic jets that resulted in extreme heating on the surface of the model at the point of impingement. The temperature history was measured using thin film temperature gauges arranged along the length of the cylinder.

Two model configurations were used to examine the

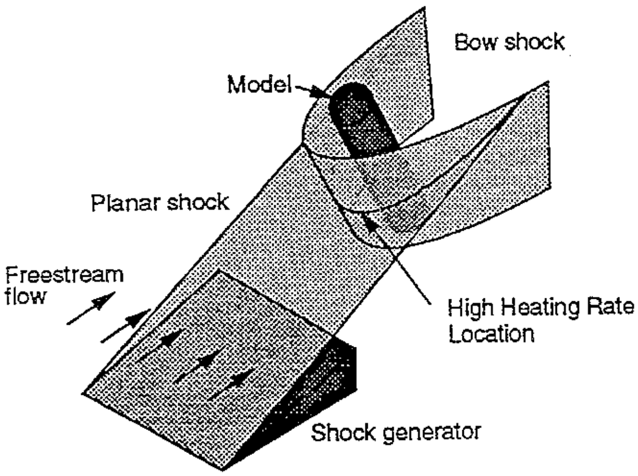

FIGURE 2: The experimental setup shows where the planar shock and the bow shock will intersect and produce the high heating rates.

lateral conduction effects and localized jet resolution. In the first case, the temperature sensors were sputtered directly onto the 1 cm diameter ceramic model. The sensors were placed as close as possible (limited by the application method) at a distance of 0.635 mm. To increase the sensor spacing on the model, a second application method was used. In this case, the sensors were sputtered onto a flat thin sheet of Upilex (a Kapton-like material). Because the surface was flat the spacing was reduced to 0.38 mm which was limited primarily by the spacing requirements of the electrical leads to the sensors. The Upilex film was then bonded to the cylindrical model with high temperature epoxy.

Approximately 90 channels of temperature measurements were collected at a sample rate of 50 Hz for each sensor for a duration of approximately 5 sec. The experiment was expected to produce a relatively constant heat flux in time, so a high sampling rate was not necessary.

SOLUTION METHODOLOGY

The inverse heat conduction solution, which is being proposed as a solution to the data reduction problem, typically attempts to identify a boundary flux or heat transfer coefficient given an interior temperature measurement. Because measurements are discrete and contain noise, the solution is usually unstable meaning that small uncertainties in the measurement result in large changes in the boundary estimate. In our case, we have a surface measurement which reduces the uncertainty in the solution somewhat compared to interior measurements. However, we must still use some optimization routine to solve the problem.

The inverse solution estimates the boundary condition that produces a calculated response matching the measured response. This methodology helps stabilize the solution because 1) we do not have to take derivatives of the experimental data, and 2) the precise matching of experimental data can be relaxed. The first point can very easily be accomplished by finding a boundary solution that minimizes the calculated and measured temperatures as in a least squares fit. The second point is realized by requiring the residual (difference between calculated and measured values) to lie within the level of measurement noise, but not to match exactly. There are two primary methodologies that will be used to perform the biased minimization.

Function-Specification

The first method examined is one that has been applied to this problem previously using several test cases with simulated data (Walker and Scott, 1997b). It predicts a value for the heat flux provided the estimate fits a predetermined functional form. We are, in essence, estimating the coefficients of a function that describe the boundary flux. Note that the estimates are found in a sequential manner such that only the current time and a "few" future times are examined simultaneously. Once the current estimate is determined, the procedure progresses to the next time step with its future measurements. By choosing the appropriate number of future time steps, we can use a first order boundary approximation without introducing overwhelming bias. The appropriate number of future times for our case is one, the minimum required to fully define a line.

At this point we have described the typical one dimensional inverse solution technique. However, the goal is to enhance the solution methodology to include multi-dimensional conduction. Therefore, we must describe the implementation of the *Function-Specification* method in the spatial direction. Again, a functional form of the boundary flux (which is now a function of location as well as time) must be predetermined, and the coefficients that minimize the objective function must be found. Since the spatial direction is elliptical, rather than parabolic, in nature the inclusion of neighboring sensors in the estimation procedure will help introduce a bias into the solution much like the future time steps did for the temporal direction. However, the solution can not march in the spatial direction as it did in the temporal direction. As a result, all spatial estimates for a given time must be found simultaneously.

The formulation for the function coefficients that make up the estimates is found from the objective function which is the sum of squares of the difference between measured ($\vec{Y}$) and calculated ($\vec{T}$) temperatures given by

$$S = (\vec{T} - \vec{Y})^{\mathrm{T}}(\vec{T} - \vec{Y}) \tag{1}$$

that is to be minimized. Here, the components of the temperature vectors consist of temperatures from different sensors at a single time followed by the temperature for the same sensors at the next time step. For example, if we use subscripts to denote spatial location and superscripts to denote time step, the temperature vectors ($\vec{T}$ and $\vec{Y}$), considering two time steps and two spatial locations, can be written as

$$\vec{T} = \{T_1^1, \quad T_2^1, \quad T_1^2, \quad T_2^2\}^{\mathsf{T}} \tag{2}$$

for a system comprised of two sensors and two measurements. Note that the calculated temperatures are found from any legitimate conduction solution.

To minimize this objective function, the derivative with respect to the flux can be set equal to zero. The boundary condition is introduced into the formulation by writing the first order Fourier series expansion of the temperature as

$$\vec{T} = \vec{T_o} + \mathbf{X}\vec{q_o} \tag{3}$$

where $\mathbf{X}$ is called the sensitivity matrix; its components are given by $X_{ij} \equiv \partial T_{oi}/\partial q_{oj}$. Note that since the conduction problem is non-linear due to temperature dependent properties and multiple conduction layers, a correction (Δq to the current estimate (q_o) was sought. Now the estimate is expressed as $\vec{q} = \vec{q_o} + \Delta q$ where the estimate at the current iteration is represented by $\vec{q_o}$. After manipulation, the flux correction can be expressed as a linear set of equations given by

$$\mathbf{X}^{\mathsf{T}}\mathbf{X}\vec{\Delta q} = \mathbf{X}^{\mathsf{T}}(\vec{Y} - \vec{T_o}) \tag{4}$$

where $\vec{T_o}$ is the calculated temperature for the current estimate.

The sensitivity matrix in the formulation for the solution (Eq. (4)) is simply found by perturbing the estimate of the current iteration and recording the temperature change. However, the perturbation that is performed must be chosen with care. As described in a previous paper (Walker and Scott, 1997b), the sensitivity "patch" determines, in some respect, the amount of bias that is introduced into the solution. In other words, where and how the current estimate is perturbed, can affect the stability and accuracy of the solution significantly.

Two different type of patches were used for this analysis. The linear patch and the constant patch correlate to a linear function specification and a constant specification method in time respectively. An example of the linear patch is shown in Fig. 3 where the extent of the patch is at the minimum of one sensor on each side. Note that to increase the bias, the span of the patch can be stretched to include two sensors on each side. In this case, a linear distribution from one (at the center) to zero (two sensors away from the center) would constitute the perturbation. Note that the estimate obtained for the sensor in question (the sensor in the center) must be distributed over the patch region in accordance with the weighting dictated by the patch. The second patch, illustrated in Fig. 3, is a piecewise constant function. The added bias can be increased, in this case as well, by expanding the extent of the patch to cover more sensors.

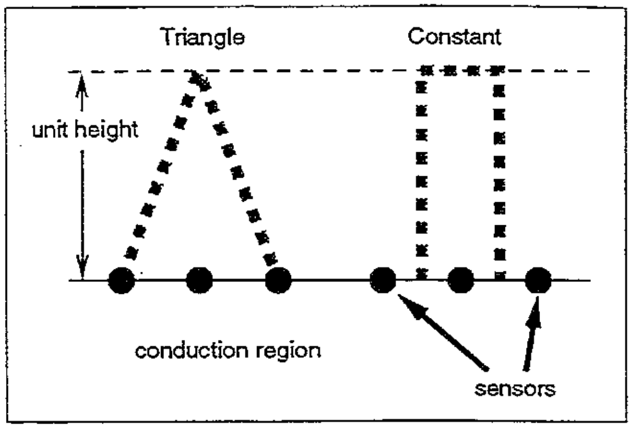

FIGURE 3: The triangular patch provides a linearly varying function in space for sensitivity calculations, whereas the constant patch provides simple implementation but too much bias.

Regularization

The *Regularization* method is a minimization problem where the difference between the temperature measurements and a calculated temperature (residual) is minimized as for the *Function-Specification* method. However, the formulation is derived from a biased least squares fit of the measured ($\vec{Y}$) and calculated ($\vec{T}$) temperature history data, (called the objective function S). The objective function is given as

$$S = (\vec{T_o} - \vec{Y})^{\mathsf{T}}(\vec{T_o} - \vec{Y}) + \alpha\mathbf{H}^{\mathsf{T}}\mathbf{H}\vec{q_o} \tag{5}$$

where the bias $\alpha\mathbf{H}^{\mathsf{T}}\mathbf{H}\vec{q_o}$ is a function of the current flux estimate being optimized (to be discussed later) and $\vec{T_o}$ contains the calculated temperatures as a result of the flux estimate at the current iteration. For this formulation, the temperature must be written in its Fourier expansion taking only the first order term, and the flux is represented in terms of a correction as before. The linear set of equations that represents the flux correction is

$$[\mathbf{X}^{\mathsf{T}}\mathbf{X} + \alpha\mathbf{H}^{\mathsf{T}}\mathbf{H}]\vec{\Delta q} = \mathbf{X}^{\mathsf{T}}(\vec{Y} - \vec{T_o}) - \alpha\mathbf{H}^{\mathsf{T}}\mathbf{H}\vec{q_o}. \tag{6}$$

This methodology is also used in a sequential manner.

The regularization that is added as bias to stabilize the solution is usually first order meaning that the type of bias forces the estimates towards a line. This means that the coefficients of the regularization term ($\mathbf{H}$) look like a difference formulation of a first derivative. The regularization parameter, α, is chosen so there is enough bias to stabilize the problem without destroying the solution with too much deterministic error. The selection of the parameter is accomplished, with trial and error, by requiring S to be on the same order as the noise in the data.

Modifications of this method were suggested by Walker and Scott (1997b) which are implemented in this work. It was suggested that because the measurements are on the surface, future time steps are unnecessary. To provide bias, however, more than a single time step must be examined. The approach, then, is to examine the previous flux and the current flux in the regularization term, and to ignore the temperature difference at the previous time step in the residual term. The problem has been simplified since the correction is only at a single time step.

To implement this method for a multidimensional problem, we must look at several adjacent sensors simultaneously. For the patch in this case, though, we will use the configuration that introduces the least bias so that this can be controlled by the regularization. The patch chosen was the linear patch that spans a single sensor on either side. Now the multidimensional approach has two separate levels of regularization that could be added, the first being regularization in space and the second being regularization in time. Therefore, the flux correction can be found using the linear equations as

$$[\mathbf{X}^T\mathbf{X} + \alpha_s \mathbf{H}_s^T \mathbf{H}_s + \alpha_t]\vec{\Delta q} = \mathbf{X}^T(\vec{Y} - \vec{T_o}) - \alpha_s \mathbf{H}_s^T \mathbf{H}_s \vec{q_o} - \alpha_t(\vec{q_{oi}} - \vec{q_{i-1}}) \quad (7)$$

where the subscript s and t represent the spatial and temporal regularization respectively. Note that the the temporal regularization is written in terms of the previous *known* heat flux. The regularization in the spatial direction is a finite difference representation of the first derivative of flux (see Scott and Beck (1987) for a complete description on the designation of first order regularization).

RESULTS

Four experimental runs are examined here, each case represents the same experimental setup in that the sweep angle of the instrumented cylinder in the flow stream is $\lambda = -25°$ as documented by Berry and Nowak (1996). Two runs (14 and 36) use a Macor model while runs 58 and 60 use the Upilex model with a high sensor resolution. A typical transient response of the sensor that receives the largest heating is indicated by the temperature shown in Fig. 4. A sudden jump in heating rate corresponds to the steep rise in temperature as the shock develops.

It should be noted that the heat flux incident on both models (Macor and Upilex) should be similar because the flow and resulting heat rates are independent of the model and material properties. However, there is some uncertainty in the properties and geometries of the materials being used. For the Upilex case, the glue layer was ignored as a conducting layer for this analysis so that the results could be compared to the one dimensional estimates performed previously which also ignore this layer. As a result, the estimate, which depends on the material properties, could contain some bias as a result of the inaccurate model. Therefore, we should not expect exact agreement between the Macor and Upilex models.

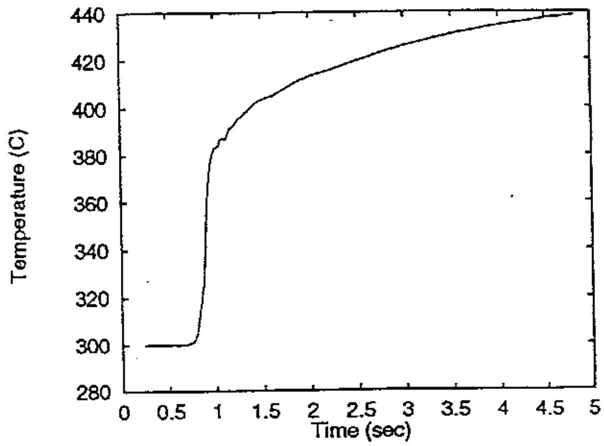

FIGURE 4: Temperature response at the sensor of peak heating for run 14 (Macor model)

The two-dimensional and one-dimensional solution techniques both appear to be dependent on the fact that the physical model can be analyzed as if it were a flat plate. In fact, this type of analysis does not hold; an accurate analysis would have to account for the effects of curvature on the conduction. Since the one-dimensional technique can not possibly account for the curvature, the two-dimensional model was restricted to a flat plate as well. (Note that Buttsworth and Jones (1997) suggested a correction factor for curved models which was not included in this analysis). Even though the analysis does not include the effects of curvature on the estimates, the effects for both methods should be identical. Therefore, a comparison between the methods is valid despite the apparent inconsistency.

At two seconds (when the shock is considered steady), the spatial distribution of the instantaneous heat flux estimates are compared. Run 14 and run 36 in Fig. 5 and Fig. 6 respectively, show the increased estimate of the two dimensional model over the one dimensional case. The validity of this finding is supported by CFD experiments (Prabhu, 1994). We find that the size of the jet is thought to be close to the spacing of the sensors. Therefore, it is difficult to determine the shape and actual magnitude of the peak heating region since we may only have a maximum of two sensors that fall in this region. Furthermore, we can not control where the jet lies relative to the sensors; it was shown by Walker and Scott (1997b) that the location of the jet relative to the sensor is significant in being able to resolve the shape of the jet. It should be noted that the distribution of the two estimates represent the same amount of integrated energy in the model.

Despite uncertainties related to the sensor spacing, we are able to glean additional knowledge about the two dimensional method vs. the one dimensional method. The flow structure near the impingement indicates a series of expansion and compression waves moving away from the

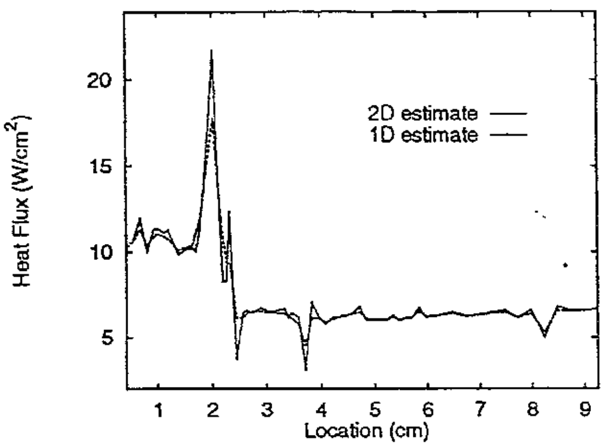

FIGURE 5: Comparison of two dimensional and one dimensional estimate of Run 14 (Macor model) at 2 sec.

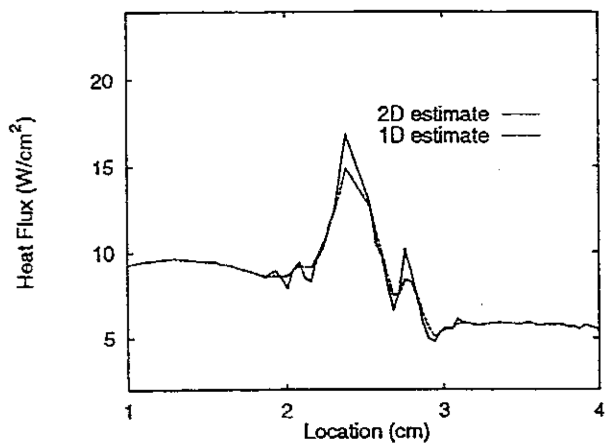

FIGURE 7: Comparison of two dimensional and one dimensional estimate of Run 58 (Upilex model) at 2 sec.

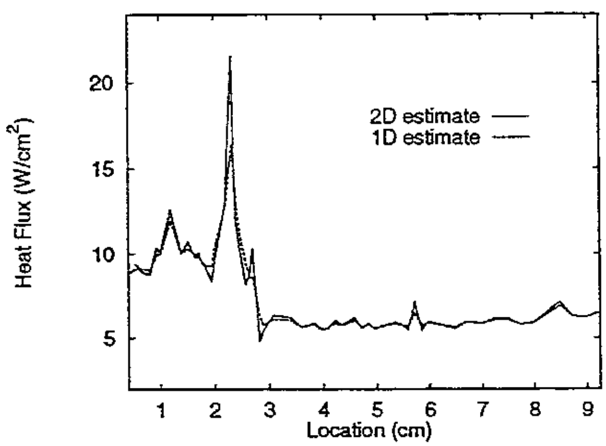

FIGURE 6: Comparison of two dimensional and one dimensional estimate of Run 36 (Macor model) at 2 sec.

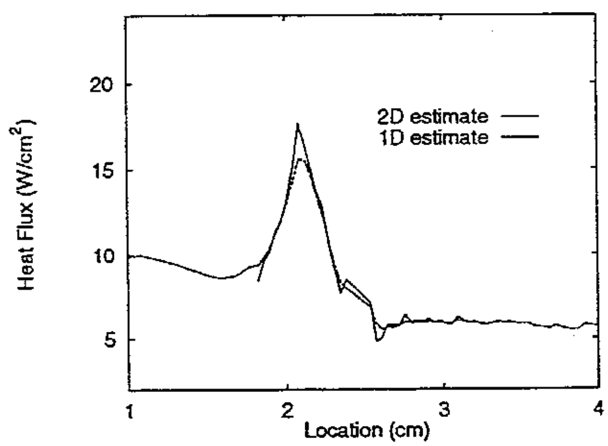

FIGURE 8: Comparison of two dimensional and one dimensional estimate of Run 60 (Upilex model) at 2 sec.

impingement site. As a result we would expect to see regions of oscillating heat flux away from the peak location. Because the two dimensional method can include conduction effects the oscillating heating rates can be recovered adjacent to the peak. At the peaks and valleys, a 20% increase/decrease in the flux estimate is seen by incorporating lateral conduction effects.

The results of the two runs with slightly higher sensor resolution (Upilex model) provide similar information to that of the Macor model. Due to the decreased sensor spacing and the different thermal properties on the surface, the Upilex runs required use of the *Regularization* method which allows the bias to be controlled more closely. The two dimensional estimates from the Upilex model are again higher in the peaks and lower in the valleys as shown in Fig. 7 and Fig. 8. For these cases, however, we are finding only a 15% increase/decrease in the estimate at the peaks and valleys respectively. For all cases, realize that where the lateral conduction is minimal (where the estimate appears constant in space), the difference between the one and two dimensional approaches is insignificant. This suggests that the two dimensional method is modeling the lateral conduction effects, and that the effect is real, not a mathematical fabrication.

Notice that the two Macor runs resulted in higher estimates than the two Upilex runs. This finding is consistent with that reported by Berry and Nowak (1996). However, it has not been determined what caused this variation. Possible explanations for the discrepancy are 1) an unknown factor in the conduction model such as the glue layer thickness or property value, 2) an unforeseen feature of the difference

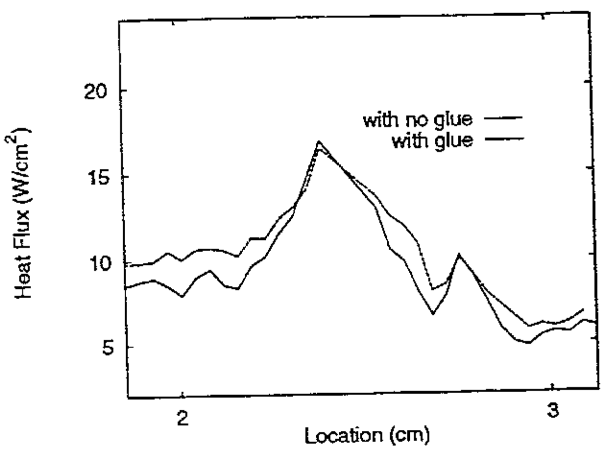

FIGURE 9: For run 58, the glue layer was initially ignored and then included into the model.

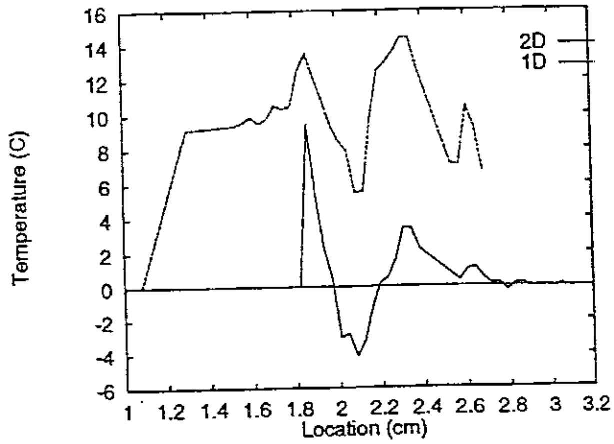

FIGURE 10: The residual is the difference between the measured and calculated temperatures for run 60 at 2 sec.

in the measurement methodology or 3) the location of the sensor relative to the fluid jet.

In an attempt to address the first contingency of whether or not the modeling of the glue layer affects the estimate, run 58 was analyzed using a numerical model with glue and one without glue. As expected in Fig. 9, the estimate is higher when the glue is considered. Note, however, that the properties of the glue tend to smooth the peak. Since we expect the estimates of the Macor model and the Upilex model to be similar, the model which ignores the glue layer appears to model the physics more accurately.

The second point suggests that one possible explanation can be gleaned from an examination of the material properties. Because the diffusivity of Macor is 3.5 times that of Upilex, we can expect the penetration depth of the Upilex to be much smaller than that of the Macor. Therefore, the temperature gradients in the sensor could be more pronounced in the Upilex case resulting in a somewhat errant reading. Therefore, the experimental setup was not designed to measure steep gradient along the sensor.

As far as the third possibility is concerned, we have explored the scenario of errors introduced because of the sensor spacing relative to the jet we are attempting to characterize. Since the spacing is different for the two runs and because the location of the jet relative to the sensors can not be determined, we can expect different levels of errors introduced here.

In an additional attempt to justify the discrepancy and to validate the estimates, the conduction solutions to the one and two dimensional estimates were found. These temperature responses were then compared to the original measurement to obtain residuals. Shown in Fig. 10 are the residuals for run 60 at a time of 2 sec. A positive residual means that the estimate overshot the target temperature. Ideally we would like the residuals to lie close to zero and within the measurement noise. It is immediately clear that the one dimensional method introduces a great deal of error into the estimate, whereas the two dimensional tends to match the measurements better.

TABLE 1: The RMS residual of the estimates at 2 sec

	2D	1D
run 58	4.74	93.2
run 60	9.98	94.9
run 14	2.13	17.6
run 36	2.18	17.4

To obtain a quantification of how well the estimation methods performed, we calculate the RMS residual. This is simply a measure of the average of the difference between the measured and calculated temperatures. Table shows a striking difference between the one and two dimensional estimates for all models. The differences between the residuals of the Upilex and Macor models are a result of the discrepancy mentioned previously, but are partly due to the fact that a different conduction model had to be employed. The varying material properties cause the sensitivity of the problem to change. Ultimately, this results in less confident solutions as demonstrated by the larger residuals.

CONCLUSIONS

Lateral conduction within the model should be considered when temperature gradients exist between adjacent sensors. When the heating rate is large and localized, the energy that diffuses laterally will normally be ignored by one dimensional estimation techniques. The multi-dimensional inverse technique demonstrates its ability to account for direct heating as well as diffusive heating by estimating a significantly higher peak heating rate. Estimation of the shape and true magnitude of the jet is limited by the sensor spacing which is thought to be the same size as the jet being measured. The estimation could, therefore, be improved by with a higher resolution gauge spacing.

More tests should provide a reliable estimate of the effects of lateral conduction between a one and two dimensional estimation approaches at the peak heating location. The methodology presented here is not limited to two dimensions. It is expected that a full three dimensional conduction model with the inverse technique will demonstrate even more decisively the fallacy of applying a one dimensional model to this data reduction problem and the effects of lateral conduction on data reduction routines.

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