HEAT SOURCE LOCALIZATION SENSITIVITY ANALYSES FOR AN ULTRASONIC SENSOR ARRAY

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ABSTRACT
State estimation procedures using the extended Kalman filter are investigated for a transient heat transfer problem in which a heat source is applied on one side of a thin plate and ultrasonic pulse time of flight is measured between spatially separated transducers on the other side of the plate. This work is an integral part of an effort to develop a system capable of locating the boundary layer transition region on a hypersonic vehicle aeroshell. Results from thermal conduction experiments involving one-way ultrasonic pulse time of flight measurements are presented. Uncertainties in the experiments and sensitivity to heating source location are discussed. Comparisons of heating source localization measurement models are conducted where ultrasonic pulse time of flight readings provide the measurement update to the extended Kalman filter. Two different measurement models are compared: 1) directly using the one-way ultrasonic pulse time of flight as the measurement vector and 2) indirectly obtaining distance from the one-way ultrasonic pulse time of flight and then using these obtained distances as the measurement vector in the extended Kalman filter. For the direct model, the Jacobian required by the extended Kalman filter is obtained numerically using finite differences from the finite element forward conduction solution. For the indirect model, the derivatives of the distances with respect to the state variables are obtained in closed form. Heating source localization results and convergence behavior are compared for the two measurement models. Two areas of sensitivity analyses are presented: 1) heat source location relative to sensor array position, and 2) sensor noise. The direct measurement model produced the best results when considering accuracy of converged solution, ability to converge to the correct solution given different initial guesses, and smoothness of convergence behavior.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>thermal diffusivity (m²/s)</td>
</tr>
<tr>
<td>C_p</td>
<td>specific heat (J/kg K)</td>
</tr>
<tr>
<td>E</td>
<td>Young’s modulus (GPa)</td>
</tr>
<tr>
<td>G</td>
<td>ultrasonic time of flight (s)</td>
</tr>
<tr>
<td>h</td>
<td>convection heat transfer coefficient (W/m² K)</td>
</tr>
<tr>
<td>k</td>
<td>thermal conductivity (kg/m³)</td>
</tr>
<tr>
<td>L</td>
<td>length (m)</td>
</tr>
<tr>
<td>q''</td>
<td>heat flux (W/m²)</td>
</tr>
<tr>
<td>T</td>
<td>temperature (°C)</td>
</tr>
<tr>
<td>t</td>
<td>time (s)</td>
</tr>
</tbody>
</table>

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Knowledge of where air flowing across a body transitions from laminar flow to turbulent flow can provide numerous benefits to air vehicle design, thermal protection system design, and air vehicle in-flight control [1]. The mechanisms leading to transition are still poorly understood [2]. At the transition between these two flow regimes, a change in body-surface temperature has been measured for a hypersonic vehicle [3]. Thus, a measurement system is envisioned that leverages the hypersonic body-surface heating profile to locate the boundary layer transition region. Ultrasonic pyrometry has proven effective for gases, fluids, and solids as long as direct access to the material where the temperature being measured is available [4] [5]. Furthermore, ultrasonic pyrometry has been used in many process control systems [6]. Ultrasonic measurements have also been used in non-destructive evaluation and defect detection for decades with a great deal of success.

For a boundary layer localization system using ultrasonic sensors, the sensors would be located on the inside surface of the aeroshell away from the harsh external conditions. Consequently, the phenomenon that is being measured is not disturbed and the sensor is not exposed to deleterious environments. In addition, the sample rate is limited only by the speed of sound through the medium, and the body-surface temperature is proportional to an easily measured quantity, time of flight. The objective of this and follow-on work is to develop a method to locate and characterize the heat flux change induced by the boundary layer transition. The solution involves a forward conduction solution and an inverse procedure. Previous work developed the forward conduction solution and compared six measurement models for the inverse procedure [7]. The work presented here focuses on the sensitivity to heating source location, the sensitivity to noise, and the ability to localize a heating source using a one-way ultrasonic pulse sensor array.

### Table 1. Material Properties for the Stainless Steel 316L Test Sample Used in the Conduction Experiments.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>density ($\rho$)</td>
<td>8,000 kg/m³</td>
</tr>
<tr>
<td>thermal conductivity ($k$)</td>
<td>14.6 W/mK</td>
</tr>
<tr>
<td>specific heat ($c_p$)</td>
<td>500 J/kg K</td>
</tr>
<tr>
<td>sound speed ($v_0$)</td>
<td>5,100 m/s @ 293 K</td>
</tr>
<tr>
<td>ultrasonic TOF temperature factor ($\xi$)</td>
<td>$110 \times 10^{-6}$/K</td>
</tr>
<tr>
<td>sample length</td>
<td>61 cm</td>
</tr>
<tr>
<td>sample width</td>
<td>30.5 cm</td>
</tr>
<tr>
<td>sample height</td>
<td>0.635 cm</td>
</tr>
</tbody>
</table>

This work concentrates first on a large flat plate heated over a small area with a known heat source. Consider a 61cm x 30.5cm x .635cm stainless steel 316L plate (Figure 1) with constant properties (Table 1). The plate is sized such that the plate edges do not affect the temperature profile in the plate during the experiment. The heating source, a Research, Inc. SpotIR 4150 heater with focusing cone, is positioned approximately 2mm from the plate surface such that its beam strikes a fixed position on the plate and is applied at $t = 300$ sec and removed at $t = 600$ sec. A parameter estimation study concluded the SpotIR heater has a heating profile of $q'' = 0.930$ MW/m² over 0.635 cm diameter circular area with a secondary heating modeled as a Gaussian with a profile of $q''_k = 100$ W/m² and a variance of $\sigma_{k}^2 = 0.0009$ m² [7]. The study also concluded the convection coefficient on the plate sides is $h = 3.20$ W/m²K. The convection coefficient on the plate edges is assumed to be $h = 3$ W/m²K and radiation effects are assumed to be negligible.

Two, 2 MHz direct deposit transducers using Ferroperm Piezoceramics Pz46 are attached to the non-heated side of the plate. The direct deposit transducers are applied 1 cm diameter and 1 mm thick. With plate center being the origin and the x-axis being the length (Figure 2), transducers are attached at (x,y) locations of (−4cm, 0cm) and (4cm, 0cm) on the non-heated side (z=0.635cm). One transducer transmits ultrasonic pulses while the other transducer receives the pulses and time of flight is recorded. Separate experiments are conducted with the source positioned on the heated side of the plate at (x,y) locations of (0cm, 0cm), (0cm, 2cm), (0cm, 4cm), (0cm, 6cm), (0cm, 8cm), and (0cm, 10cm). Flat black paint is applied to a 1.5 cm wide strip at the plate center to maximize energy absorption from the heater. The plate is oriented vertically with the positive y-axis pointing up. Data acquisition equipment employing cross-correlation techniques is used to determine and record ultrasonic pulse time of flight readings once per second during the experiment.

\[ v \quad \text{sound speed (m/s)} \]
\[ \theta \quad \text{temperature change relative to reference (K)} \]
\[ \xi \quad \text{ultrasonic time of flight temperature factor (1/K)} \]
\[ \rho \quad \text{density (kg/m³)} \]
\[ \sigma^2 \quad \text{variance for a Gaussian probability density function} \]

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[527x597] $m K$ 14 6 W/mK 110 10^-6 1/K 61 cm 30.5 cm 0.635 cm

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The forward conduction solution used in the sensitivity study and the heating source localization inverse procedure was developed in previous work [7] and leverages COMSOL Multiphysics® version 3.5 by the COMSOL Group and MATLAB® by The Mathworks, Inc. The model uses parameters identified in the previous section in the COMSOL® 3D, heat transfer, conduction, transient analysis module. The solution uses a finite element mesh with smaller elements near the heat source and larger elements near the plate edges to conserve computing resources. A grid convergence study was performed to ensure grid independence. Both the number of elements in the plate’s $x-y$ plane and the number of layers in the plate’s thickness were considered. The grid convergence study led to the selection of three mesh layers through the plate’s thickness dimension, 9,780 total elements, and 45,983 degrees of freedom.

The measured time of flight is related to the average temperature between the transducers by [8] [7]

$$G_{ij} = \frac{R_{ij}}{v_0} \left(1 + \xi \theta_{avg} \right)^i$$

where $R_{ij}$ is the distance between transducers, $v_0$ is the sound speed in the material at a reference temperature, $\xi$ is the ultrasonic time of flight factor which is material dependent (Table 1), and $\theta_{avg}$ is the change in temperature from the reference temperature. Since $R_{ij}$ is known with insufficient accuracy to compare the time of flight from the model to the measured values, the time of flight can be normalized to the initial state.

$$G_{ij} = \left(1 + \xi \frac{T_{avg} - T_0}{R_{ij}} \right) = 1 + \xi \left(\frac{T_{avg} - T_0}{R_{ij}} \right)$$

where $G_0$ is the average time of flight recorded from $t = 1$ to 299 seconds before the heater is turned on. Figures 3 and 4 illustrate the agreement between the COMSOL® model and the ultrasonic time of flight measured during the experiment. Agreement between the model and the experiment is acceptable, however the magnitude with the heat source at $(0\text{cm}, 0\text{cm})$ and $(0\text{cm}, 2\text{cm})$ is underestimated by the model. Figure 5 illustrates the time of flight measurements during the beginning part of the experiment and highlights the response time for the various sensor distances.

**SENSITIVITY TO SOURCE LOCATION**

To illustrate the one-way pulse method’s sensitivity to source location, it is necessary to examine how the average temperature between the two sensors is affected by the relative position of source. Note that this discussion assumes the source is
FIGURE 3. COMPARISON OF THE COMSOL® MODEL WITH THE ONE-WAY ULTRASONIC PULSE EXPERIMENT WITH HEAT SOURCE LOCATED BETWEEN THE SENSORS (TOP CURVE) AND OFFSET BY 2CM, 4CM, 6CM, 8CM, AND 10CM. MODEL USES TEMPERATURES ALONG THE NON-HEATED SURFACE OF THE PLATE.

FIGURE 4. RESIDUALS BETWEEN THE COMSOL® MODEL AND THE ONE-WAY ULTRASONIC PULSE EXPERIMENT. MODEL USES TEMPERATURES ALONG THE NON-HEATED SURFACE OF THE PLATE.

FIGURE 5. ONE-WAY ULTRASONIC PULSE TIME OF FLIGHT MEASUREMENTS FOR THE BEGINNING PART OF THE HEATING PHASE.

not moving with time. Figure 6 illustrates the temperature profile on the plate’s non-heated side at \( t = 320 \) s with the source at (0cm, 0cm). The average temperature between the two sensors is 13.5 K. If the source was at (2 cm, 0cm) as in Figure 7, the average temperature is nearly identical at 13.2 K. We conclude, then, that even with knowledge that the source is in-between the sensors, its \( x \) location cannot be determined very accurately. If the source were further to the right, say at (3 cm, 0), the average temperature would be 11.8 K indicating that sensitivity to \( x \) location is greater close to the transducers. If the source was instead offset in the \( y \) direction at (0, 1 cm) as in Figure 8, the average temperature is 5.3 K. Thus, the sensitivity to source position in the \( y \)-direction is greater than for the \( x \)-direction for this sensor pair.

The temperature gradient is expressed as

\[
|\nabla \theta_{\text{avg}}| = \sqrt{\left(\frac{\partial \theta_{\text{avg}}}{\partial x}\right)^2 + \left(\frac{\partial \theta_{\text{avg}}}{\partial y}\right)^2}
\] (3)

Figures 9 and 10 illustrate \( |\nabla \theta_{\text{avg}}| \). These figures highlight the high sensitivity regions around the sensor path and the drastic drop-off near the path. These figures support the observation above that sensitivity is greater perpendicular to the ultrasonic propagation path. One should notice the rapid decrease in sensitivity as the source location nears the path between sensors.
FIGURE 6. TEMPERATURE RESPONSE AT \( t = 320 \) S WITH SOURCE AT \((0,0)\). \( \theta_{\text{avg}} = 13.5 \) K BETWEEN SENSORS.

FIGURE 7. TEMPERATURE RESPONSE AT \( t = 320 \) S WITH SOURCE AT \((0,1 \text{ CM})\). \( \theta_{\text{avg}} = 5.3 \) K BETWEEN SENSORS.

FIGURE 8. TEMPERATURE RESPONSE AT \( t = 320 \) S WITH SOURCE AT \((2 \text{ CM},0)\). \( \theta_{\text{avg}} = 13.2 \) K BETWEEN SENSORS.

FIGURE 9. TEMPERATURE GRADIENT FOR ONE-WAY PULSE SENSOR CONFIGURATION AND ALL POSSIBLE HEATING SOURCE LOCATIONS AT \( T = 320 \) S.
HEATING SOURCE LOCALIZATION

This section examines heating source localization using four ultrasonic transducers in an 8 cm square pattern. Data from the ultrasonic pulse experiments above are used to simulate this sensor grid.

Locating and characterizing the boundary layer transition depends upon many factors such as heating source movements in time, heating source magnitude changes in time, and other transient behaviors. Fairly restrictive assumptions can be imposed that simplify the problem. Analysis and algorithm development can proceed using these restrictive assumptions and then assumptions can be relaxed in stages to achieve the end result of source localization and characterization. The assumptions for this work are:

1. Source in fixed position (location unknown)
2. Source applied at time $t = 300$ sec and removed at $t = 600$ sec
3. $q = 0.930 \text{MW/m}^2$ over $0.00635 \text{m}$ diameter circular area while source applied (value obtained in parameter identification above)
4. Secondary heating is characterized by a Gaussian with magnitude $q_g = 100 \text{W/m}^2$ and variance $\sigma_g^2 = 0.0009 \text{m}^2$ while source applied
5. Convection coefficient $h = 3.20 \text{W/m}^2\text{K}$ on both sides of the plate (value obtained in parameter identification above)
6. Convection coefficient $h = 3 \text{W/m}^2\text{K}$ on the plate edges
7. Thermal conductivity $k = 15 \text{W/mK}$
8. Specific heat $C_p = 500 \text{J/kgK}$ and density $\rho = 8,000 \text{kg/m}^3$
9. Positions of sensors are ($\pm 4 \text{cm}$, $\pm 4 \text{cm}$) on the non-heated side (Figure 11)

The two measurement models analyzed in this work are:

1. Ultrasonic pulse one-way time of flight measurement model
2. Ellipse from ultrasonic one-way pulse time of flight measurement model

Comparison of the two measurement models is performed using the extended Kalman filter (algorithm in Table 2) to locate the source $(x_q, y_q)$. For both measurement models, the state is $X_t = [x_q, y_q]^T$ and there is no input to the state thus the state model is $a = I_2$ and the state Jacobian is $A = I_2$. Sensitivity of the state variance was compared for values from $\sigma^2 = 0.01 \text{m}^2$ to $0.000001 \text{m}^2$ with the lower values providing a damping effect. A state variance of $\sigma^2 = 0.0001 \text{m}^2$ provides a good compromise between damping and stability and will be used for all measurement model comparisons in this work. Thus, the state covariance matrix is $Q_t = 0.0001 \text{m}^2 \times I_2$, where $I_2$ is a $2 \times 2$ identity matrix.

Ultrasonic Pulse One-way Time of Flight Measurement Model

This direct measurement model consists of obtaining expected temperatures from the COMSOL® model, computing the average temperature between the transducers, and then computing an expected time of flight to form $a(U_i, X_{i-1})$ (equation 4). For the current analysis, the average temperature is based on the line on the non-heated plate surface between the two sensors.
The Jacobian partial derivatives are obtained using time of flight difference when moving the source in the $x$ and $y$ directions independently (equation 5).

$$b(\overline{x}_t) = \begin{bmatrix} G_1 \\ G_2 \\ G_3 \\ G_4 \end{bmatrix}$$

$$B_t = \begin{bmatrix} -\frac{\partial G_1}{\partial x_i} & -\frac{\partial G_1}{\partial y_i} \\ -\frac{\partial G_2}{\partial x_i} & -\frac{\partial G_2}{\partial y_i} \\ -\frac{\partial G_3}{\partial x_i} & -\frac{\partial G_3}{\partial y_i} \\ -\frac{\partial G_4}{\partial x_i} & -\frac{\partial G_4}{\partial y_i} \end{bmatrix}$$

where $t$ is time in seconds with a time step of 1 second, $\overline{G}_i$ with $i = 1, 2, 3, 4$ is the ultrasonic pulse time of flight with the heating source located at $(x_i, y_i)$, and $(x_s, y_s)$ with $i = 1, 2, 3, 4$ are the locations of four transducers. The Jacobian $B_t$ is constructed using the derivatives with respect to sensor position for convenience since this information can be obtained with one COMSOL® simulation. The derivatives are obtained from the COMSOL® model using finite differences by independently varying the $x$ and $y$ positions of all sensors by 0.0001 m. Based on the flat plate experiment above, non-dimensional sensor noise is assumed be $\pm 6 \times 10^{-3}$ and is normally distributed ($\sigma^2 = ((6 \times 10^{-5})/3)^2 = 4 \times 10^{-10}$). Solution instabilities using this variance were solved by increasing the variance to $4 \times 10^{-7}$ which effectively dampens the solution and prevents large changes from one iteration to the next. The measurement covariance matrix, therefore, is $R = 4 \times 10^{-7} \ast I_4$.

**Ellipses From Ultrasonic Pulse One-way Time of Flight Measurement Model**

In this indirect model, a particular ultrasonic pulse time of flight means that the source could be anywhere on an elliptical shape around the sensors. Figure 12 illustrates the geometry of an ellipse. The two sensors are assumed to be the focus points for the ellipse. Since the distance between sensors is known, ellipse parameters $c$ and $d$ can be related to each other and the ellipse can be represented with just one parameter $c$.

$$r_{is} + r_{js} = 2c = \sqrt{r_{ij}^2 + 4d^2}$$

where $i$ and $j$ are sensors and $s$ is heat source.

$$c = \frac{1}{2} \sqrt{r_{ij}^2 + 4d^2} = \frac{r_{is} + r_{js}}{2}$$

$$r_{is} = \sqrt{(x_i - x_s)^2 + (y_i - y_s)^2}$$

$$r_{js} = \sqrt{(x_j - x_s)^2 + (y_j - y_s)^2}$$

$$B_t = \begin{bmatrix} \frac{\partial u}{\partial x_i} & \frac{\partial u}{\partial y_i} \\ \frac{\partial u}{\partial x_j} & \frac{\partial u}{\partial y_j} \\ \vdots & \vdots \\ \frac{\partial u}{\partial x_s} & \frac{\partial u}{\partial y_s} \end{bmatrix}$$

$$\frac{\partial c}{\partial x_i} = \frac{1}{2} \begin{bmatrix} x_i - x_s \\ r_{is} \end{bmatrix}$$

$$\frac{\partial c}{\partial y_i} = \frac{1}{2} \begin{bmatrix} y_i - y_s \\ r_{is} \end{bmatrix}$$

The parameter $c$ is measured indirectly by first measuring the one-way ultrasonic pulse time of flight. The forward conduction solution is used to get time of flight for a range of $c$ values. The $c$ parameter is analogous to radius and is the orthogonal distance from the ultrasonic path between two sensors and the source at $s$. 

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**TABLE 2. EXTENDED KALMAN FILTER ALGORITHM.**

<table>
<thead>
<tr>
<th>Step</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\overline{x}<em>t = a(U_t, \overline{x}</em>{t-1})$</td>
</tr>
<tr>
<td>2</td>
<td>$\Sigma_t = A_t \Sigma_{t-1} A_t^T + Q$</td>
</tr>
<tr>
<td>3</td>
<td>$K_t = \Sigma_t B_t^T (B_t \Sigma_t B_t^T + R_t)^{-1}$</td>
</tr>
<tr>
<td>4</td>
<td>$\overline{x}_t = \overline{x}_t + K_t (Z_t - b(\overline{x}_t))$</td>
</tr>
<tr>
<td>5</td>
<td>$\Sigma_t = (I - K_t B_t) \Sigma_t$</td>
</tr>
<tr>
<td>6</td>
<td>Return to step 1 if solution not converged</td>
</tr>
</tbody>
</table>
where \( t \) is time in seconds with a time step of 1 second, \( \overline{G}_{i} \) with \( i = 1, 2, 3, 4 \) is the ultrasonic pulse time of flight with the heating source located at \((x_j, y_j)\), and \((x_i, y_i)\) with \( i = 1, 2, 3, 4 \) are the locations of four transducers. The Jacobian \( B_t \) is constructed using the derivatives with respect to sensor position for convenience since this information can be obtained with one COMSOL® simulation. The derivatives are obtained from the COMSOL® model using finite differences by independently varying the \( x \) and \( y \) positions of all sensors by 0.0001m. Based on the flat plate experiment above, sensor noise is assumed be \( \pm 1.05 \times 10^{-8} \) sec and is normally distributed \( (\sigma^2 = 1.22 \times 10^{-17} \text{sec}^2) \). The sensor noise in terms of temperature can be expressed as

\[
\theta_{\text{noise}} = \frac{G_{\text{noise}} v_0}{L_5^{2}} = 6.09 \text{K}
\]

(15)

Using the average slope of 0.015 m/K determined in previous work [7], ellipse noise for the \( c \) parameter from ultrasonic pulse time of flight measurement model is \( \pm 0.0914 \) m and is normally distributed \( (\sigma^2 = 9.28 \times 10^{-4} \text{m}^2) \). Since the measurement covariance matrix \( R \) represents the measurement noise of the \( c \) parameter, the measurement covariance is \( 3.19 \times 10^{-10} \text{m}^2 \) \( ([1.05 \times 10^{-8} \text{sec} / 3 \times 5,100 \text{m/sec sound speed}]^2) \).

**Extended Kalman Filter Convergence Behavior**

Extended Kalman filter convergence behavior for both measurement models are compared in Figures 13 through 18. With the heating source located inside the sensor grid (Figure 13), the one-way ultrasonic pulse time of flight measurement model converges to the correct location while the ellipse model does not. With the heating source located at the edge of the sensor grid (Figure 14), the one-way ultrasonic pulse time of flight measurement models once again converges to the correct location. With the heating source located outside of the sensor grid (Figure 15), neither measurement model converges to the correct location. These examples started with an initial guess of \((0 \text{ cm}, 0 \text{ cm})\) for the heating source location. Figures 16 through 18 illustrate the convergence behavior for both models using an initial guess of \((8 \text{ cm}, 8 \text{ cm})\). Interestingly, the initial guess does not affect the outcome when the heating source is located at the edge of the sensor grid, but the initial guess plays a significant role in the outcome when the heating source is inside or outside of the sensor grid.

Figure 19 illustrates the sensitivity to sensor noise for the one-way pulse time of flight measurement model. The experiment was simulated with the COMSOL® model using dimensionless noise comparable to the noise recorded during the experiments \((6e-5)\) and double and triple the recorded noise \((12e-5 \text{ and } 18e-5)\). The convergence behavior demonstrates the robustness inherent in the solution. Sensitivity to sensor noise for the ellipse model is being considered in ongoing research.

Repeating the experiment using a complete sensor grid of four transducers instead of simulating the sensor grid with separate experiments using two transducers might produce different convergence behavior, especially for heating source locations outside the sensor grid.

**CONCLUSIONS**

Results were presented from forward conduction solution development and flat plate experimentation with a known heat source. The extended Kalman filter convergence behavior was compared using two ultrasonic pulse one-way time of flight measurement models. The one-way ultrasonic pulse time of flight measurement model (the direct model) produced the best results.
FIGURE 14. EXTENDED KALMAN FILTER CONVERGENCE FOR BOTH ONE-WAY ULTRASONIC PULSE MEASUREMENT MODELS WITH SOURCE AT (4 CM, 0 CM) AND INITIAL GUESS OF (0 CM, 0 CM).

FIGURE 15. EXTENDED KALMAN FILTER CONVERGENCE FOR BOTH ONE-WAY ULTRASONIC PULSE MEASUREMENT MODELS WITH SOURCE AT (6 CM, 0 CM) AND INITIAL GUESS OF (0 CM, 0 CM).

FIGURE 16. EXTENDED KALMAN FILTER CONVERGENCE FOR BOTH ONE-WAY ULTRASONIC PULSE MEASUREMENT MODELS WITH SOURCE AT (2 CM, 0 CM) AND INITIAL GUESS OF (8 CM, 8 CM).

FIGURE 17. EXTENDED KALMAN FILTER CONVERGENCE FOR BOTH ONE-WAY ULTRASONIC PULSE MEASUREMENT MODELS WITH SOURCE AT (4 CM, 0 CM) AND INITIAL GUESS OF (8 CM, 8 CM).
when considering accuracy of converged solution, ability to converge to the correct solution given different initial guesses, and smoothness of convergence behavior. Whereas this work had no inputs to the state model, the ability to add inputs to a recursive state estimator (e.g., a Gaussian filter) is anticipated to be more robust for heat source localization and in turn for boundary layer transition localization and characterization.

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