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A NEW TECHNIQUE FOR HEAT FLUX DETERMINATION

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ABSTRACT

A new method for estimating heat fluxes from heating rate measurements and an approach to measure heating rates is proposed. Heating rate is defined as the time rate of change of the temperature. The example problem involves analytic heat conduction in a one-dimensional slab, where the measurement location coincides with the location of the estimated heat flux. The new method involves the solution to a Volterra equation of the second kind, which is inherently more stable than Volterra equations of the first kind. The estimates of the new approach are compared to typical inverse solution methods. The heating rate measurements are accomplished by leveraging the temperature dependent decay rate of thermographic phosphors. Results indicate that the new data-reduction method is far more stable than minimizing temperature residuals with errors of the order of the measurement noise.

INTRODUCTION

Aerospace vehicles often encounter deleterious heating environments during high-speed flight. Accurate characterization of heating loads, then, is crucial to survivability of aerospace structures. In controlled tunnel tests, prediction of a heat flux incident on a test article can provide meaningful information concerning the environment that a full-scale structure will experience, whereas temperature measurements typically do not scale well from tunnel to flight conditions. Herein lies the necessity to predict heat fluxes. However, characterization of heating loads (or heat flux) in tests is not trivial because direct measurement is difficult.

Although the prediction of heat fluxes contain inherent challenges, three methods have evolved to accomplish the task with varying degrees of success. The most obvious approach to predicting heat fluxes is direct measurement with a heat flux gauge [1, 2]. These gauges usually consist of a thermopile and actually measure temperature differences, which can be converted to heat fluxes. Problems associated with these measurements include slow response time, flow disturbance and calibration. A second approach involves measurement via a calorimeter [3]. Like the heat flux gauge, these devices often suffer from slow response time and flow disturbance. Further, the estimation of the heat flux from the actual measurement is complicated by the fact that the heat load is not uniform across a surface because of lateral heat conduction [4]. Recently new techniques for heat flux determination have become more popular such as thermochromatic liquid crystals (TLC) [5]. Conceptually, this process involves layers of crystals in thin films aligning themselves based on temperature gradients. The orientation can be inferred from spectral measurement of reflected light. The technique is global but currently suffers from lagging response times [6] and a limited range of operation.

An attractive alternative for predicting heat fluxes is direct measurement of temperature followed by an

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data reduction technique to estimate the incident flux [7]. Temperature measurement can be accomplished with tremendous precision and accuracy [8] with high frequency components [9]. In fact thin film temperature measurements have become so robust and used extensively to predict heat fluxes that the literature will often incorrectly refer to these types of measurements as heat flux measurements. However, the data reduction is inherently unstable [10] and multi-dimensional effects are difficult to resolve [11]. Inverse methods, however, address these concerns with statistically based estimation methods. For a comprehensive evaluation of one-dimensional methods for surface temperature methods, see Walker and Scott [12]. The advantages of temperature measurement devices coupled with inverse techniques make this approach to heat flux determination attractive. For example, because temperature measurement devices are usually smaller, the time response is much better and the effect on the incident flows can be minimized. In addition, temperature measurements are easier to calibrate and the data reduction is not limited to one-dimensional estimation [13]. Therefore, it can be argued that temperature measurements and inverse data reduction techniques are preferable.

Despite advances in techniques devised to solve ill-posed problems and account for noisy data, the fact remains that appropriate data reduction remains a balancing act between introducing smoothing bias and amplification of noise. Solutions in many cases still contain unacceptable errors [14]. The present work suggests that many of the stability problems associated with the inverse heat conduction problem can be mitigated by measuring a different quantity, namely the heating rate. The heating rate in the present context is defined as the time rate of change of temperature for a given location and time. It can be shown that the data reduction is inherently more stable if this quantity could be measured. However, no method currently exists to measure the heating rate directly. This approach represents a departure from typical heat flux determination methods such as those discussed briefly above, because the temperature is not explicitly required for the estimation of heat flux.

The objectives of the present work are to demonstrate a stable method for estimating heat flux from measured heating rate and to describe a technique to measure heating rate. The estimation component involves a test problem with various boundary conditions and simulated noise, which is a common approach to evaluating inverse methods. The measurement technique involves the temperature sensitive decay rate of thermographic phosphors (TGP). Although TGPs have been used to measure temperature (particularly for remote measurement), the current approach will leverage particular properties of TGPs to obtain measurements, which are proportional to the heating rate, not temperature.

Thermographic phosphors are rare-earth-doped ceramics that fluoresce when exposed to ultraviolet radiation or similar excitation. In general, the intensity, frequency line shift and decay rate are all temperature dependent. As a result, they have been used for remote temperature sensing in many applications [15]. Many materials have been used and tuned for specific applications with a great deal of success [16, 17, 18, e.g.]. However, they have never been used to predict a heating rate. It is the strong dependence of the decay rate on temperature that will be leveraged to acquire a heating rate. This simple proof-of-concept described herein demonstrates the ability to extract heat fluxes with far greater accuracy than previously possible.

THEORY

Although the approach presented here can be generalized to almost any conduction problem, the following example will be used for illustration purposes. Assume one-dimensional conduction in a slab of length L. The governing equation for temperature with boundary conditions is given as

$$\frac{\partial^2\theta}{\partial\eta^2} = \frac{\partial\theta}{\partial\xi}; \quad (1)$$

$$-\left.\frac{\partial\theta}{\partial\eta}\right|_{\eta=0} = Q(\xi); \quad (2)$$

$$\theta(\eta = 1) = 0; \quad (3)$$

$$\theta(\xi = 0) = 0. \quad (4)$$

where the temperature has been normalized to the initial temperature T_o such that $\theta = T - T_o$. The spatial and temporal coordinate have been non-dimensionalized (i.e. $\eta = x/L$, $\xi = \alpha t/L^2$, where α is the thermal diffusivity). The heat flux Q at $\eta = 0$ is a continuous function of ξ and is presumed known. Using an integral transform technique, the infinite series solution is found to be

$$\theta(\eta,\xi) = \sum_{m=1}^{\infty} 2\cos(\beta_m\eta)\int_0^{\xi} Q(\xi')e^{\beta_m^2(\xi'-\xi)}\,d\xi', \quad (5)$$

where the eigenvalues are found as the positive roots of $\cos(\beta_m) = 0$ ($\beta_m = (2m+1)\pi/2$ where $m = 1,2,3,\ldots$). A more computationally efficient or convenient solution may be found, but for the present discussion, the foregoing expression exhibits the requisite traits.

If the heat flux $Q(\xi)$ is unknown, equation 5 is a Volterra equation of the first kind for a known temperature, and the solution is unstable for discrete temperature measurements with noise. If we assume that that the temperature on the surface is measured at N discrete times, then an estimate for the unknown heat flux can be found using standard inverse solution techniques. A brief description of a discrete least-squares residual-minimization technique follows.

First we represent the unknown heat flux as a truncated series

$$Q(\xi) \approx Q_d(\xi) = \sum_{i=1}^{M} a_i \Phi_i(\xi), \tag{6}$$

where Q_d is an infinite-term series of linear independent basis functions (Φ) and expansion coefficients (a_i). $\xi -> \xi_i$ has been implicitly assumed. Note that $M \leq N$, where the equality will lead to a unique solution (exact matching of data), and the inequality will represent an over determined system. Discrete surface temperature calculations can be constructed by replacing the continuous heat flux $Q(\xi)$ in equation 5 with the approximate heat flux (equation 6). The residual to be minimized is constructed from the difference between the measurements (Y_i) and the calculated values of temperature at the surface.

$$R_Y = Y_i - \int_0^{\xi} Q_d(\xi') K(\xi_i, \xi')\, d\xi', \tag{7}$$

where equation 5 is written in terms of a kernel and the discrete boundary heat flux. The kernel is simply the temperature solution scaled by the discrete flux. For the present example the kernel is given as

$$K(\xi, \xi') = \sum_{m=1}^{\infty} 2\cos(\beta_m \eta) e^{\beta_m^2 (\xi' - \xi)}. \tag{8}$$

A least-squares minimization of the L_2-norm of the residual leads to an estimation of the expansion coefficients. The global matrix solution follows [19] and is given as $\mathbf{A}\bar{a} = \bar{b}$, where $\bar{a}$ is the vector of unknown expansion coefficients, and

$$\bar{b}_i = \sum_{j=1}^{N} \theta_j P_i(\xi_i), \text{and} \tag{9}$$

$$\mathbf{A}_{ij} = \sum_{j=1}^{N} P_k(\xi_j) P_i(\xi_j). \tag{10}$$

In the foregoing solution,

$$P_k(\xi_j) = \int_0^{\xi_j} \Phi_k(\xi') K(\xi_j, \xi')\, d\xi'. \tag{11}$$

This formulation represents a classical approach to solving inverse heat conduction problems with measured temperatures.

In the present work, heat flux is calculated analytically by assuming a piecewise constant heat flux over each time step and setting the residual in Eq. 7 to zero. The integration can be performed analytically leading to a set of N equations

$$Y_i = \sum_{r=2}^{i} Q_r \sum_{m=0}^{\infty} \left[e^{\beta_m^2 (\xi_r - \xi_i)} - e^{\beta_m^2 (\xi_{r-1} - \xi_i)} \right]. \tag{12}$$

The solution of the foregoing expression leads to an estimate of the heat flux at each time step. Realize that the solution contains a slight bias depending on whether the constant, Q_r, is assumed to be over the previous or future time step. However, this bias can be essentially eliminated by choosing an average value for the heat flux. In other words, we can replace Q_r with $(Q_r + Q_{r-1})/2$.

The alternate approach to predicting heat flux requires measurement and calculation of the heating rate. A heating rate can be found analytically by differentiating equation 5 with respect to time. The formulation, given as

$$\Phi(\xi) = \frac{\partial \theta}{\partial \xi} = \sum_{m=0}^{\infty} 2\cos(\beta_m \eta) \left[Q(\xi) - \beta_m^2 \int_0^{\xi} Q(\xi') e^{\beta_m^2 (\xi' - \xi)}\, d\xi' \right], \tag{13}$$

suggests that the nature of the solution for heat flux is not as ill-conditioned because equation 13 is a Volterra equation of the second kind [20]. The solution for heat flux follows the solution from a measured temperature (i.e. equation 7) by considering a finite series of basis functions (equation 6). Now the residual is given as the difference between the measured heating rate (H_i) and the calculated heating rate (equation 13). As before, the heat flux can be calculated analytically by assuming a functional form of the heat flux over the time step that is integrable. It is not immediately clear that the infinite series in Eq. 13 converges in a finite number of terms. For high-order terms, the heat flux must equal the integral on the right hand side. In fact, the convergence rate is sensitive to the functional form of the heat flux approximation between times. Therefore, this feature precludes the most simple solution approach of piecewise constant heat flux over the time step because this approximation creates a biased estimator that requires many terms to converge.

A satisfactory approach that generates unbiased solutions in a reasonable number of terms is a piecewise linear approximation.

$$Q(\xi) = Q_{r-1} + \frac{Q_r - Q_{r-1}}{\xi_r - \xi_{r-1}}(\xi - \xi_{r-1}), \qquad \xi_{r-1} < \xi < \xi_r. \tag{14}$$

As before, the residual is set equal to zero, $R_H = H_i - \Phi(\xi_i) = 0$, and the heat flux is calculated analytically from the set of N equations.

$$\Phi_i = \sum_{r=2}^{i}\sum_{m=0}^{\infty} 2\cos(\beta_m \eta) \left\{ \frac{e^{\beta_m^2(\xi_{r-1}-\xi_i)}}{\beta_m^2(\xi_{r-1}-\xi_i)}\left[Q_{r-1}(1-\beta_m^2\xi_{r-1}+\beta_m^2\xi_r) - Q_r\right] + \frac{e^{\beta_m^2(\xi_r-\xi_i)}}{\beta_m^2(\xi_{r-1}-\xi_i)}\left[Q_r(1+\beta_m^2\xi_{r-1}-\beta_m^2\xi_r) - Q_{r-1}\right]\right\}. \tag{15}$$

It is implicitly assumed that $\Phi(\xi_i) = \Phi_i$. The solution to the foregoing expression is unbiased and stable, which will be demonstrated.

The measurement of heating rate is not a direct measurement. Instead, the intensity of phosphor emission is measured at a sample rate that is higher than the decay rate. The phosphors are excited in pulses that are longer than the decay rate, so the measurements give an effective change in intensity with respect to time for a pulse. This appears to be similar to measuring two temperatures in time and differentiating to obtain a heating rate. However, the change in intensity is an exponential function, and the measurement is not differentiated. Therefore, measurement noise is not amplified significantly during the conversion to a heating rate.

Because the phosphor emission intensity is an exponential function in time,

$$\frac{I}{I_o} = \exp\left[-\frac{t}{\tau}\right], \tag{16}$$

the decay rate can be estimated from a series of intensity measurements. If we assume that the phosphor has been completely and carefully characterized, the decay rate is a material property that is a well-known function of temperature. This is the approach used to predict temperature from phosphor-decay rate. For interesting engineering problems, though, the temperature is not constant. Therefore, we use a first-order Taylor series expansion of the decay rate to introduce the derivative of τ. Now the normalized intensity,

$$\frac{I}{I_o} = \exp\left[-\frac{t}{\tau + \frac{\partial\tau}{\partial t}\Delta t}\right], \tag{17}$$

contains two parameters that are estimated from a series of intensity measurements. Now the heating rate can be found from

$$\frac{\partial\theta}{\partial t} = \frac{\partial\theta}{\partial\tau}\frac{\partial\tau}{\partial t}, \tag{18}$$

where $\partial\theta/\partial\tau$ is a temperature dependent material property. This approach requires three intensity measurements compared to two temperature measurements required for a finite-difference heating rate data collection. However, the integration process in Eq. 17 is more stable than a differencing of noisy data [21].

RESULTS

To compare different methods for predicting heat flux, a known analytic function was chosen as the exact heat flux, Q, to which all estimates will be compared. Table 1 lists each type of heat flux that was examined. The exact analytic solution to both the temperature, Y, and the heating rate, H, (Eqs. 5 and 13, respectively) were calculated to provide the measurement data, from which the estimates will be derived. Further, normally distributed random noise was added to the measurements to evaluate how the estimators behave when measurement error exists in the temperature, Y_n, and heating rate, H_n. Initially, the estimates were obtained from the discrete measurements by assuming a piecewise constant heat flux over each time step. This essentially reduces the basis to be unity over the time step and zero everywhere else.

The first test case examined is the triangular heat flux, whose surface temperature history is shown in Fig. 1 with and without additional noise. The noisy signal is obtained by adding a normally distributed random component with standard deviation of $\sigma = 0.01$. Visually, the noise is a small percentage of the actual signal. The simulated discrete temperature measurement is sampled at 50 Hz. The two

Table 1. Reported errors are root mean square of the differences between the exact heat flux, Q, and the estimated heat fluxes.

	Q_Y	Q_{Y_n}	Q_H	Q_{H_n}	Q_{H_d}
zero	n/a	0.4732	n/a	0.0021	0.0443
triangle	0.0329	0.4458	~ 0	0.0021	0.0451
square	0.2929	0.2923	~ 0	0.0021	0.0956
sinusoid	0.1300	0.5894	0.0189	0.0185	0.0808

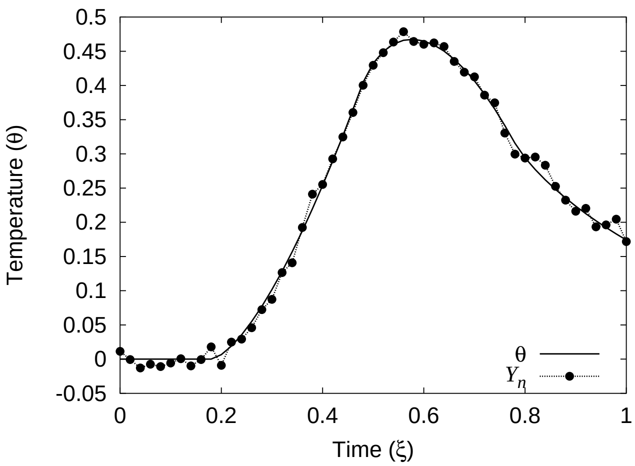

Figure 1. The exact temperature response to a triangular heat flux profile (see Fig. 2). The noisy signal has a normally distributed random noise with standard deviation $\sigma = 0.01$.

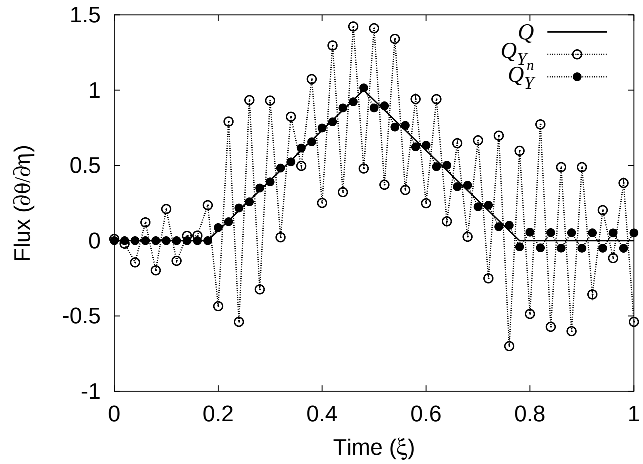

Figure 2. The heat flux history estimates are derived from the temperatures in Fig. 1. The estimate from the exact temperature, Q_Y generates errors because of the piecewise-constant approximation. The estimate from noisy data, Q_{Y_n}, demonstrate how small errors become amplified.

temperature histories are used to predict heat fluxes by inverting Eq. 5. This is inherently an unstable process, and the estimate from noisy data, Q_{Y_n} in Fig. 2, shows that small errors become amplified in the solution. The errors seen in the solution from exact data arise from the piecewise-constant approximation. No attempt to relax the solution, introduce bias or otherwise implement any inverse technique has been made. Therefore, the results represent an exact matching or zero residual technique, which is a "worst-case" scenario. Nevertheless the estimates serve as a benchmark for subsequent comparison.

The new approach requires a measured heating rate, which is generated similarly to the measured temperature and is shown in Fig. 3. The exact solution to the heating rate equation (Eq. 13) with a known triangular heat flux is sampled at 50 Hz to generate the exact measurement, H. Further, normally distributed random noise with a standard deviation of $\sigma = 0.01$ is added to create noisy signal, H_n. These data are displayed in Fig. 3 along with a finite difference formulation of the heating rate, H_d. The difference scheme represents a crude method for heating rate determination. In other words, H_d is generated by applying a central difference scheme to the noisy temperature measurements, Y_n. This approach produces the overwhelmingly noisy signal in Fig. 3. The heat flux estimates are then calculated directly by inverting Eq. 13, which is identical to the approach for estimating heat fluxes from temperature measurements. The results shown in Fig. 4 demonstrate that the inversion of the Volterra equation of the second kind is not nearly as sensitive to noise as the Volterra equation of the first kind as expected. In fact the error in the noisy estimate, Q_{H_n} appears to be damped, and the solution contains no bias and almost no noise. However, the surprising feature is that Q_{H_d}, which was produced from a signal whose noise is nearly comparable to the signal, faithfully reproduces the original heat flux with reduced errors. Table 1 demonstrates that the estimates from all varieties of the heating rate (H, H_n and H_d) contain very little error.

It is important to note that the heating rate estimator is a biased estimator. Even though the errors in the measurements are damped, the solution is sensitive to the approximation made for the heat flux between time steps. The simplest approach is to assume piecewise constant. However, this approximation introduces significant bias and requires an infinite number of terms to converge. Further, round-off issues become significant. As a result, a piecewise

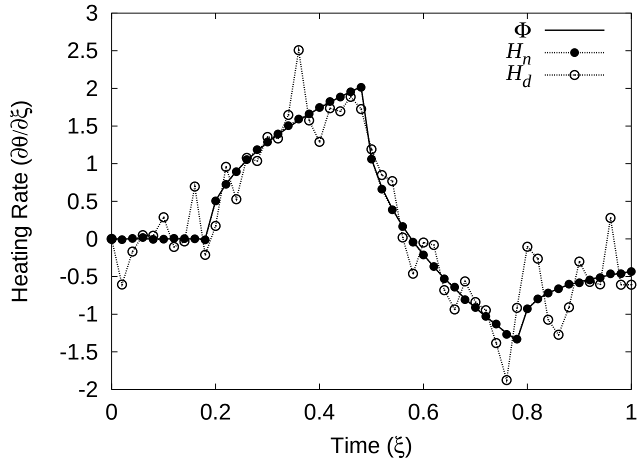

Figure 3. Heating rate response to a triangular flux. H is the exact solution; H_n contains normally distributed random noise with a standard deviation of $\sigma = 0.01$; H_d is a central finite difference of the noisy temperature measurement Y_n.

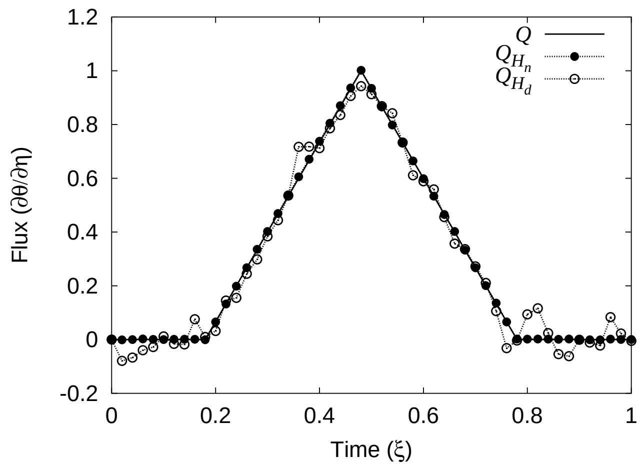

Figure 4. The heat flux history estimates are derived from the heating rate for a triangular heat flux (see Fig. 3). The errors are damped for both cases and any bias is imperceptible.

linear distribution of heat flux was assumed. Because the original heat flux can be exactly represented as a set of piecewise linear segments, the solution does not contain any approximation and converges in less than 20 terms. It is not clear what the requirement for convergence actually is, so the approach was tested on a square heat flux as shown in Fig. 5. At the corners, the exact flux actually *is* piecewise constant, not piecewise linear, so we might expect to see a bias. Visually, the estimate from noisy data, Q_{H_n}, exhibits

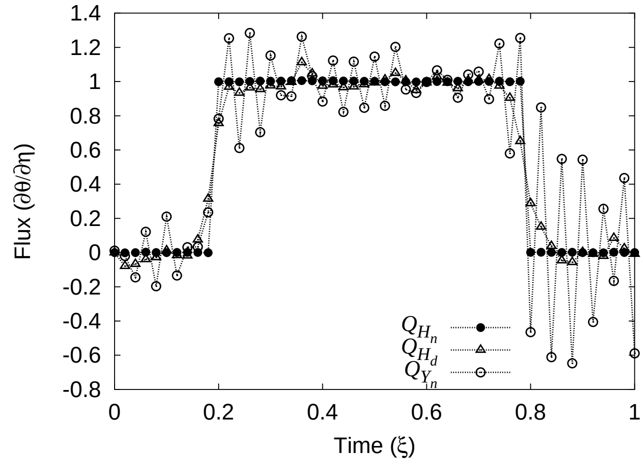

Figure 5. Heat flux estimates of a square flux. The exact heat flux is not shown because it is virtually identical to Q_{H_n}.

no bias, and measurement errors are damped. In fact the exact flux is not shown in Fig. 5 because the graphical resolution required to distinguish the two estimates is not adequate.

CONCLUSIONS

A new approach to predicting heat flux is proposed, which will improve heat flux estimates by reducing instabilities inherent in temperature to heat flux data reduction methods. By measuring the heating rate, the integral equation for heat flux becomes a Volterra equation of the second kind, which is inherently more stable than the first kind. Analysis confirms that the method is a great deal more stable and can accommodate more noise than an approach that uses temperature measurements. The method for measuring heating rate uses thermographic phosphors, which is already being used to measure temperatures. The method should be evaluated with experimental data to verify the utility of this method.

In general the data reduction method described is valid for temperatures measured in any location, even though in the present treatment the measurement is assumed to be on the surface. A surface treatment was assumed because the heating rate measurement process described can only occur at the surface. Additional work will evaluate the methodology for interior temperature measurements.

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