**TRANSPORT INVOLVING CONDUCTING FIBERS IN A NON-CONDUCTING MATRIX**

R. A. Hansel¹, D. G. Walker¹², J. Rozen³

---

**Introduction**

- Thermal and electrical transport through a low-conductivity matrix containing high-conductivity fibers are important to several applications including flexible thin-film transistors (FET), proton exchange membranes (PEM), and direct-energy conversion devices.
- In direct energy conversion devices high electrical conductivity and low thermal conductivity are preferred for superior performance. However, most materials do not exhibit both of these properties simultaneously and strategies for tuning the material properties are being sought.
- Nanofibers can limit the thermal transport through phonon confinement and boundary scattering while maintaining high electrical conductivity.
- The net result is a material with high electrical conductivity and low thermal conductivity.

---

**Transport Models**

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix (W/mK)</td>
<td>Fiber (W/mK)</td>
</tr>
<tr>
<td>0.2</td>
<td>100</td>
</tr>
<tr>
<td>0.5</td>
<td>150</td>
</tr>
<tr>
<td>1.0</td>
<td>200</td>
</tr>
</tbody>
</table>

**Effective Medium Approximation**

- Maxwell's model for circular inclusions has been adapted for ellipsoidal inclusions:

\[ k = \frac{k_m - \varepsilon k_f}{1 + \varepsilon k_f k_m} \]

where

\[ \varepsilon = \frac{a_f^2}{a_m^2 + a_f^2} \]

- \( k_m \) is matrix material thermal conductivity
- \( \varepsilon \) is the aspect ratio of the fiber
- \( \varepsilon \) is the area fraction of fibers to matrix material
- Underestimates higher fiber densities because the model does not account for fiber overlap.

**Thermal Transport**

- Fiber network is discretized into square mesh
- Mesh size is smaller than fiber width
- Each node has 4 resistors with equivalent resistance
- A system of linear equations is used to determine resistance:

\[ \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix} \mathbf{v} = \mathbf{R} \]

- \( A \) is a square matrix that has the size of the number of sites plus the number of resistors placed in each node with equations for Ohm’s law and Kirchoff’s law
- \( v \) is the vector containing the potential at each node

**Electrical Transport**

- Matrix does not contribute to conductance
- The network of fibers conducts when a direct conduction path forms between the two contacts
- A resistor is placed between any two fiber-fiber intersections
- Resistance \( R \) depends on material resistivity \( \rho \), length, and fiber diameter which reduces to:

\[ R = \frac{2 \rho}{\pi a_f} \]

---

**Results**

- Fiber lengths are normalized to the size of the device and are randomly oriented on the device
- Effective conductivity was determined for different fiber lengths
- Vertical error bars represent variance in conductivity due to statistical variation
- Horizontal error bars represent variance in pixel density which results from fiber overlap

**Thermal Transport**

- Maxwell's model predicts lower conductivity at higher pixel density since fiber overlap is not considered
- Fibers with longer length conduct more than shorter fiber, despite similar pixel densities

---

**Conclusions**

Analytical solutions were obtained by calculating the effective conductivity of a matrix with ellipsoidal geometry using an effective medium approximation. This model works well for dilute systems with low fiber density. However, this model does not account for fiber-fiber overlap and therefore under-predicts the conductivity for systems with high fiber density.

Conductivity is higher in systems with long fibers because of higher fiber densities and higher pixel densities.

Thermal and electrical properties can be decoupled to predict material properties for fiber-polymer thin film devices and thermoelectric devices.

Decoupling transport properties would be useful for the design of thermoelectric devices which require thermal and electrical properties can be decoupled to predict material properties for fiber-polymer thin film devices and thermoelectric devices.

Useful in design of fiber-laden electronic devices.

**Lorenz Number**

- The ratio of thermal to electrical conductivity
- Our results show that the Lorenz number is not constant
- Thermal and electrical properties can be decoupled
- Useful in design of fiber-laden electronic devices

---

¹Interdisciplinary Graduate Program in Materials Science, Vanderbilt University, Nashville, TN
²Department of Mechanical Engineering, Vanderbilt University, Nashville, TN
³Institute of Nanoscale Science and Engineering, Vanderbilt University, Nashville, TN